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Growth Theory: the chain-reaction, its attractor and the United States economy

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According to Kaldor (1972), economic growth is the result of a chain-reaction between increases in supply and demand. It is the starting point of a new Keynesian and endogenous growth theory.

It is demonstrated that the steady states of this process have three main unexpected properties on the long term. Firstly, output growth rate is a linear function of employment growth rate and of net investment rate:

$$g_Y = \frac{1-\propto}{2 \propto} g_L + \frac{A}{2} i_n$$

where $\propto$ is the profit share in income, i_n the net investment rate (or the net saving rate) and A the productivity of the capacity investment. Secondly, this linear relation plays the role of an attractor during the long term, with annual trajectories (output and employment growth rates) wrapping around this relationship. A high profit share in income (more than 1/3) leads to an economic slowdown.

The theoretical lessons are consistent with the stylized facts highlighted by economists and with the reality of the U.S. economy from 1961 to 2015. These results demonstrate the interest of this new way of research.

Keywords: *chain-reaction, effective demand, endogenous growth, distribution, United States.*

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INTRODUCTION

Kaldor carried out a series of studies seeking to characterize the process of economic growth (1956, 1961, and 1972), specifically the link between this process and the principle of effective demand, accumulation of capital, increasing returns and technical progress. 'Given that factor, the process of economic development can be looked upon as the resultant of a continued process of interaction–one could almost say, of a chain-reaction–between demand increases which have been induced by increases in supply, and increases in supply which have been evoked by increases in demand', concluded Kaldor (1972, p.1246).

This vision of a chain-reaction, neglected in subsequent work on economic growth, is the foundation of a new Keynesian and endogenous growth theory we propose, building on the ideas of Schumpeter (1911, 1942), Keynes (1936), Palley (1996), Aghion-Howitt (1998), Nelson (2005) and Piketty (2014).

For Keynes, the entrepreneur makes decisions on output and employment by anticipating the supply-demand balance ("principle of effective demand"), taking into account a long-term forecast of the marginal return on capital ("marginal efficiency of capital"); the entrepreneur also seeks to minimize the risk of loss of competitiveness in the long term[2]. For Palley, increasing returns must be combined with the principle of effective demand and accumulation of capital governed by investment.

For Schumpeter, the entrepreneur is the source of creative destruction through investments to "produce more" or "produce differently"; the entrepreneur seeks to minimize the total cost per unit of output[3]. For Aghion-Howitt, growth is an endogenous process compatible with creative destruction and whose steady state is sought. For Nelson, the growth process is inherently unbalanced, the concept of equilibrium is no longer appropriate and should be replaced by an attractor, a regulator attracting paths but never fixing them permanently.

For Piketty, empirical examination of the distribution of wealth for more than two centuries shows that the return on capital is higher than the growth rate of the economy ($r < g$ described as the "fundamental law of capitalism"). There is also an interaction between the long-term profit share in income ($\propto$) and the capital/income ratio (β); furthermore, given the data, there is reason to doubt the existence of "human capital" as an output factor.

These considerations are the basis of a model of a growth process based on entrepreneurs anticipating effective demand and competitive supply. Growth is endogenous and investments induce creative destruction. By nature, reality is rarely as expected; this imbalance requires entrepreneurs to adapt, constantly reformulating new expectations in the following period, leading to an endless process of rebalancing-imbalance. The attractor then replaces the concept of equilibrium.

We define the principle of competitive supply, and then the "effective and competitive" equilibrium as the balance anticipated over a period by entrepreneurs who want to reach the balance of effective demand, while implementing competitive productive combinations. Thus,

[2] 'The output from equipment produced to-day will have to compete, in the course of its life, with the output from equipment produced subsequently, perhaps at a lower labor cost, perhaps by an improved technique... Moreover, the entrepreneur's profit (in terms of money) from equipment, old or new, will be reduced, if all output comes to be produced more cheaply.' (Keynes, 1936: p. 141).

[3] 'Everyone agrees that private and socialist managements will introduce improvements if, with the new method of production, the total cost per unit of product is expected to be smaller than the prime cost per unit of product with the method actually in use.' (Schumpeter, 1942: p. 97).

we combine the principle of effective demand and the principle of competitive supply in order to define "effective and competitive" equilibrium.

Obviously, "effective and competitive" equilibrium is never reached, without exception, and entrepreneurs must formulate a new equilibrium in the next period. We show that this growth process, consisting of a succession of effective and competitive equilibria sought by entrepreneurs, admits steady states in the long term, where expectations approach reality and growth is balanced. In fact, these steady states become an attractor.

Three unexpected theoretical lessons then appear. Firstly, output growth rate is a linear function of the employment growth rate and the net investment rate. Secondly, this relationship characterizes an attractor of the economic cycle. Thirdly, when the profit share in income becomes higher than 1/3, the output growth rate decreases.

In the first section, we develop the key elements of the growth process, modeled by an endogenous growth model of output and employment and by establishing "effective and competitive" equilibrium. In a second section, we identify the steady states and we then show the three unexpected properties. In a third section, the major theoretical lessons are compared with the reality of the United States economy from 1961 to 2015; in particular, the existence of an economic cycle attractor is shown. In a fourth section, this new theory is the subject of a discussion showing consistency with the stylized facts highlighted by Kaldor (1961), Barro and Sala-I-Martin (1995), Verdoorn (1949, 1993), and Piketty (2014).

1. THE GROWTH PROCESS: A CHAIN-REACTION TRIGGERED BY ENTREPRENEURS

The process of economic growth is the result of a chain-reaction between demand increases induced by increases in supply and increases in supply evoked by increases in demand. Each of these processes triggers the next, which is the characteristic of a chain-reaction. The latter can be boosted (economic boom) or stifled (stagnation or economic recession).

In the short term, entrepreneurs formulate rational expectations about fundamentals, taking into account a long-term view of the marginal efficiency of capital, reflecting confidence in the long-term state. They place themselves at the equilibrium of effective demand. At the same time, they make decisions to obtain the most competitive productive combinations, while taking into account the conditions prevailing in the different markets. For example, they have to decide the volume of capacity investment or rationalization investment, and the volume of jobs created or destroyed; they use simple criteria such as retaining projects with minimum total cost per unit of output, given the conditions in the labor market and the financial market.

Obviously, the expectations of entrepreneurs are rarely realized, given the great many uncertainties, their limited rationality and the unpredictable changes in many variables. However, entrepreneurs develop strategies to adapt to the new context by constantly seeking competitiveness and the balance between supply and demand.

1.1 ANTICIPATED INCREASES IN AGGREGATE SUPPLY AND DEMAND

We assume that at time t, the economy is in equilibrium from the point of view of supply Z, demand D and output Y. For entrepreneurs, investment I is then defined, as is the marginal efficiency of capital e_K, i.e. the expected long-term return on investment. Three types of investment are distinguished: capacity investment, rationalization investment and replacement investment. Capacity investment is used to "produce more", while rationalization investment is used to "produce differently".

We shall highlight the conditions of an equilibrium at time $t + dt$, taking into account the expectations formulated by entrepreneurs. To this end, the anticipated increases in aggregate supply and in aggregate demand will be determined, along with the equilibrium of effective demand. Competitive productive combinations will then be anticipated and sought after by entrepreneurs. This will highlight increases in output, employment and investment, in the "effective and competitive" equilibrium, as functions of the marginal efficiency of capital estimated by entrepreneurs. The growth process is modeled as a succession of effective and competitive equilibria.

First, we determine the expected increases in aggregate supply and aggregate demand over the interval $t + dt$.

Anticipated increase in aggregate supply

The anticipated increase in aggregate supply, represented by $\dot{Z}^a$, is the expected additional output volume corresponding to the employment increase of $\dot{L}^a$[4]:

$$\dot{Z}^a = \varphi(\dot{L}^a) \quad \text{with } Z(t) = D(t) = Y(t) \tag{1}$$

The marginal function of global supply rests on three founding hypotheses, which will be explained in turn.

- *The additional supply, as a function of capacity investment:* the additional supply $\dot{Z}^a$ is determined by the capacity investment $x^a I_n$ where I_n is the net investment volume[5]:

  $$\dot{Z}^a = Ax^a I_n \quad with \quad A = constant \qquad 0 \leq x^a \leq 1 \tag{2}$$

  $x^a I_n$ is the volume of investment that will be engaged in additional output; it will be referred to as "capacity investment". The complementary investment $(1 - x^a)I_n$ will be referred to as "rationalization investment".

  A is the "Productivity of capacity investment" (PCI), i.e. the productivity per unit of capacity investment. Henceforth, it is assumed to be constant. The PCI reflects the productivity of the investments used in the growth of output.

  x^a is the share of the net investment volume involved in additional output: it will be referred to as the "Ratio of capacity investment" (RCI), in this case the anticipated one. Any increase of x^a results in an increase in capacity of output. The RCI reflects the ability of the economy to invest in output growth.

- *Job creation, with increasing returns:* entrepreneurs create jobs according to the additional supply, the elasticity of "supply to jobs created" being variable:

  $$\frac{L_c^a}{L} = e_c^a \frac{\dot{Z}^a}{Z} \qquad e_c^a < 1 \tag{3}$$

  L_c^a is the job creation associated with capacity investment. The jobs created are more productive, given the existence of increasing returns; hence an elasticity of less than 1. The creation of jobs, expected by entrepreneurs, is thus:

  $$L_c^a = e_c^a A x^a \frac{I_n}{Y} L \qquad e_c^a < 1 \tag{4}$$

- *Destruction of jobs with capital-labor substitution*: entrepreneurs destroy jobs on the basis of the "supply shortfall" $A(1 - x^a)I_n$, the elasticity of "supply shortfall to jobs destroyed" being variable:

[4] The index *a* indicates the anticipated (or *ex ante*) character of the variable.

[5] The difference between the (gross) investment volume and the net investment volume is due to the replacement investment volume.

$$\frac{L_d^a}{L} = e_d^a \frac{A(1-x^a)I_n}{Y} \qquad L_d^a = e_d^a A(1-x^a)\frac{I_n}{Y}L \tag{5}$$

The expected increase in employment is therefore:

$$\dot{L}^a = [(e_c^a + e_d^a)x^a - e_d^a]Ai_nL \qquad -e_d^a Ai_nL \le \dot{L}^a \le e_c^a Ai_nL \qquad i_n = \frac{I_n}{Y} \tag{6}$$

The increase in employment depends on net investment rate, and expected values for RCI and elasticities. The marginal global supply function is then written:

$$\dot{Z}^a = \varphi(\dot{L}^a) = \frac{Y}{(e_c^a + e_d^a)L}\dot{L}^a + \frac{e_d^a A\, i_n Y}{(e_c^a + e_d^a)} \tag{7}$$
$$0 \le \dot{Z}^a \le AI_n \qquad -e_d^a Ai_nL \le \dot{L}^a \le e_c^a Ai_nL$$

The marginal function of aggregate supply is a linear (increasing) function of the increase in employment and is represented in Figure 1.

Anticipated increase in aggregate demand

The increase in aggregate demand, represented by $\dot{D}^a$, is the additional product that entrepreneurs hope to derive from the additional employment $\dot{L}^a$:

$$\dot{D}^a = f(\dot{L}^a) \tag{8}$$

The additional demand $\dot{D}^a$ is composed of the additional volumes that entrepreneurs and consumers should spend on consumption and investment, taking into account the expected increase in employment. The marginal propensity to consume is taken as $p_{\dot{C}}$. Whence:

$$\dot{D}^a = p_{\dot{C}}\dot{Z}^a + \dot{I}^a \tag{9}$$

The increase in aggregate demand is then written*:*

$$\dot{D}^a = f(\dot{L}^a) = p_{\dot{C}}\frac{Y}{(e_c^a + e_d^a)L}\dot{L}^a + p_{\dot{C}}\frac{e_d^a Ai_nY}{(e_c^a + e_d^a)} + \dot{I}^a \tag{10}$$

The marginal function of aggregate demand is a linear (increasing) function of the increase in employment and is represented in Figure 1.

1.2 THE PRINCIPLE OF EFFECTIVE DEMAND

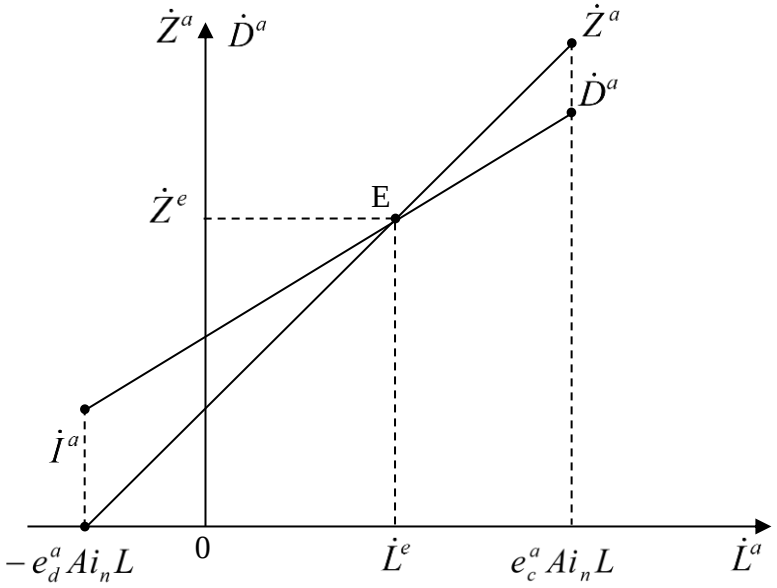

Figure 1 Increases in aggregate supply $\dot{Z}^a$ and in aggregate demand $\dot{D}^a$

"Additional effective demand" $\dot{D}^e$ is defined as the expected increase in demand at the point of intersection of increases in aggregate supply and demand, with an increase in the volume of employment $\dot{L}^e$. At the point of intersection, the anticipated profit is maximal. It can then be written:

$$\dot{D}^e = \dot{Z}^e = \dot{Y}^e \tag{11}$$

The aggregate demand curve (see Figure 1) is also a straight line that necessarily intersects the aggregate supply curve (point E). Point E, called "effective equilibrium", represents the new equilibrium anticipated by entrepreneurs:

$$\dot{Y}^e = Ax^e i_n Y \qquad \dot{L}^e = (e_c^a + e_d^a)x^e A i_n L - e_d^a A i_n L \qquad x^e = \frac{\dot{I}^a}{(1 - p_{\dot{C}})A i_n Y} \tag{12}$$

This balance imposes interdependencies between variables. For example, if entrepreneurs decide on an increase on investment and anticipate a particular marginal propensity to consume, this balance determines the RCI.

1.3 THE PRINCIPLE OF COMPETITIVE SUPPLY

At the equilibrium point of additional effective demand, entrepreneurs are looking for the most competitive productive combinations, that is to say those that both minimize the risk of loss of competitiveness against competing firms and ensure the return on investments. This leads them to determine the RCI and the elasticities according to the marginal efficiency of capital. To this end, entrepreneurs take into account three constraints: total cost per unit of output, short-term return and return independent of strategies.

The constraint of total cost per unit of output

Entrepreneurs minimize the expected total cost per unit of output[6] under a condition linked to the marginal efficiency of capital:

$$\frac{cost^e}{\dot{Y}^e} = \frac{\omega L_c^e + e_K x^e I_n}{A x^e I_n} \quad \textit{subject to} \quad \frac{\omega L_c^e}{x^e I_n} = \frac{C_1}{e_K} \quad x^e \neq 0 \quad e_K > 0 \tag{13}$$

The expected total cost of output $(cost^e)$ includes the cost of increasing employment and the cost of capacity investment. Minimization of the total cost per unit of output will be done under a condition, the cost of job creation per unit of capacity investment being inversely proportional to the marginal efficiency of capital. This condition reflects the risk that competitors will arise over the long term, this risk being all the stronger as the marginal efficiency of capital is greater; it aims to minimize the risk of loss of competitiveness in the long term.

The minimization formula is equivalent to $Min\{(1-\propto)e_c^a A + e_K\}$ under the condition $C_1^{'} = e_c^a e_K$, given the relationship $\omega L = (1-\propto)Y$ where $\propto$ is the profit share in income at time *t*. The solution is easily obtained by the substitution of the condition into the function to be minimized:

$$f(x^e, e_K) = (1-\propto)e_c^a A + e_K = (1-\propto)\frac{C_1'}{e_K}A + e_K$$
$$\frac{\partial f}{\partial e_K} = -(1-\propto)\frac{C_1' A}{(e_K)^2} + 1 = 0 \quad \frac{\partial^2 f}{\partial (e_K)^2} > 0 \tag{14}$$

The minimum[7] is such that:

$$e_c^a = \frac{e_K}{(1-\propto)A} \qquad 0 < \propto < 1 \qquad e_c^a < 1 \tag{15}$$

[6] We consider the additional cost per unit of additional output.

[7] The solution can also be obtained by writing as equal the two terms of the sum to be minimized.

This elasticity is completely defined; it is a function of the marginal efficiency of capital and of the profit share in income.

The constraint of short-term return on capital

Entrepreneurs aim to obtain, in the short term, an investment return equal to the expected return, i.e. the marginal efficiency of capital e_K:

$$\frac{\propto \dot{Y}^e}{I_n} = e_K \quad \Rightarrow \quad x^e = \frac{e_K}{\propto A} \qquad x^e \leq 1 \quad \Rightarrow \quad e_K \leq \propto A \tag{16}$$

As a result, the RCI is a function of the marginal efficiency of capital. The existence of conditions on the elasticity (equation 15) induces a condition on the profit share, which must be less than 1/2:

$$x^e = \frac{1-\propto}{\propto} e_c^a \qquad x^e = 1 \;\; and \;\; e_c^a < 1 \quad \Rightarrow \quad 0 < \propto < 1/2 \tag{17}$$

The constraint of return on capital independent of strategies

The return on investment depends on the expected increase in employment, assuming that profit share in income and wages are constant over time:

$$\frac{\propto \dot{Y}^e}{I_n} = \frac{\propto}{1-\propto} \frac{\omega \dot{L}^e}{I_n} = \propto A[(e_c^a + e_d^a)x^e - e_d^a] \tag{18}$$

In a stylized way, two strategies are possible: either a change in the RCI or a change in the elasticities. Both strategies must be equivalent in terms of return so that companies remain profitable whatever strategy is chosen. From equation (18), we obtain:

$$(e_c^a + e_d^a)dx^e = x^e de_c^a + x^e de_d^a - de_d^a \quad \Rightarrow \quad e_c^a + e_d^a = x^e \frac{de_c^a}{dx^e} + (x^e - 1)\frac{de_d^a}{dx^e} \tag{19}$$

Taking into account equations 15 and 16, one obtains:

$$\frac{\propto}{(1-\propto)} x^e + e_d^a = \frac{\propto}{(1-\propto)} x^e + (x^e - 1)\frac{de_d^a}{dx^e} \quad \Rightarrow \quad e_d^a + (1 - x^e)\frac{de_d^a}{dx^e} = 0 \tag{20}$$

$$e_d^a = u(1 - x^e) \qquad u = constant$$

As a result, the optimum shape of the elasticity related to jobs destroyed is a decreasing function of the RCI. In addition, entrepreneurs expect the same return on capital associated with pure rationalization investment (RCI equal to 0) and pure capacity investment (RCI equal to 1). The expected profit for pure rationalization investment is equal to the reduction in the wage bill, in view of the loss of jobs and stagnation of wages. Whence:

$$(1-\propto)uA = \propto A \quad \Rightarrow \quad e_d^a = \frac{\propto}{(1-\propto)}(1 - x^e) = \frac{\propto A - e_K}{(1-\propto)A} \tag{21}$$

Ultimately, the elasticity related to jobs destroyed is a function of the marginal efficiency of capital and of the profit share in income[8].

1.4 EFFECTIVE AND COMPETITIVE EQUILIBRIUM

The "effective and competitive" equilibrium is defined as that of the effective demand anticipated by the entrepreneurs and supported by a competitive supply. The RCI and the elasticities are now functions of the marginal efficiency of capital. Entrepreneurs, after having defined the marginal efficiency e_K, are able to determine the increase in output $\dot{Y}^{ec}$, the increase in employment $\dot{L}^{ec}$ and the increase in investment $\dot{I}^{ec}$:

$$\dot{Y}^{ec} = \frac{e_K}{\propto} i_n Y \qquad \dot{L}^{ec} = \frac{2e_K - \propto A}{1-\propto} i_n L \qquad \dot{I}^{ec} = (1 - p_{\dot{C}})\frac{e_K}{\propto} i_n Y \tag{22}$$

[8] The sum of the two elasticities is a constant.

The result is that the effective and competitive equilibrium is uniquely defined and that the growth rates of output, employment and investment at equilibrium are expressed in terms of the marginal efficiency of capital:

$$\frac{\dot{Y}^{ec}}{Y} = \frac{e_K}{\propto} i_n \qquad \frac{\dot{L}^{ec}}{L} = \frac{2e_K - \propto A}{1 - \propto} i_n \qquad \frac{\dot{I}^{ec}}{I} = \frac{(1 - p_{\dot{C}})}{(1 - p_C)} \frac{e_K}{\propto} i_n \tag{23}$$

where p_C is the mean propensity to consume. These equations are valid for a non-zero marginal efficiency of capital. On the other hand[9]:

$$e_K = 0 \qquad x^{ec} = 0 \qquad \dot{Y}^{ec} = 0 \qquad \dot{L}^{ec} = -\frac{\propto A}{1 - \propto} i_n L \qquad \dot{I}^{ec} = 0 \tag{24}$$

At the effective and competitive equilibrium, a remarkable linear equation links the output growth rate, the employment growth rate and the net investment rate:

$$\frac{\dot{Y}^{ec}}{Y^{ec}} = \frac{1 - \propto}{2 \propto} \frac{\dot{L}^{ec}}{L^{ec}} + \frac{A}{2} i_n \tag{25}$$

Thus, the output growth rate is a linear function of both the employment growth rate and the net investment rate, the coefficients being a function of the profit share in income and of the PCI respectively. This equation is independent of the RCI, which makes it a relationship independent of the economic cycle.

We will now make a simplifying hypothesis, generally observed in reality over a certain period: the mean propensity to consume p_C and the proportion of replacement investment δ are assumed to be constant, which makes it possible to write:

$$p_C = p_{\dot{C}} = constant \qquad I_n = (1 - \delta) I = (1 - \delta)(1 - p_C) Y \tag{26}$$

As a result, the net investment rate is also constant:

$$i_n = \frac{I_n}{Y} = (1 - \delta)(1 - p_C) = constant \tag{27}$$

The fundamental equations become:

$$\frac{\dot{Y}^{ec}}{Y} = \frac{\dot{I}^{ec}}{I} = \frac{e_K}{\propto} i_n \qquad \frac{\dot{L}^{ec}}{L} = \frac{2e_K - \propto A}{1 - \propto} i_n \tag{28}$$

2. ON THE LONG TERM, THE STEADY STATES AND THE INSIGHTS

The process of economic growth is modeled by a succession of effective and competitive equilibria anticipated by entrepreneurs. In order to identify the stationary states of this process in the long term, we assume that the expectations of entrepreneurs are satisfied in reality and that long-run growth is balanced[10]. It is shown here that, in the long term, the stationary states are steady states, the growth rates of output and employment being constant over time. Three remarkable insights are deduced.

2.1 THE STEADY STATES

The expected values of the fundamental variables meet the reality:

$$\dot{Y}^{ec} = \dot{Y} \qquad \dot{L}^{ec} = \dot{L} \qquad x^{ec} = x \qquad \dot{I}_n^{ec} = \dot{I}_n \tag{29}$$

By definition (see Equation 16), the marginal efficiency of capital is equal to the marginal return on capital q:

$$e_K = \propto Ax = \frac{\propto \dot{Y}}{I_n} = q \tag{30}$$

[9] It is not possible to minimize the expected total cost per unit of output.

[10] Following the line of studies by Harrod (1939, 1948), Domar (1947), and Aghion-Howitt (1998).

It is now assumed that growth is balanced: the growth rate of output is equal to that of capital ("guaranteed" growth rate). In other words, the mean productivity of capital is constant over time. Whence:

$$\frac{\dot{Y}}{Y} = \frac{\dot{K}}{K} \Leftrightarrow \frac{Y}{K} = \frac{\dot{Y}}{\dot{K}} = constant = \frac{q}{\propto} = Ax \tag{31}$$

For steady states, the RCI is constant along with the marginal return on capital, the return on capital r and the capital/income ratio:

$$x = constant \qquad r = \propto \frac{Y}{K} = \propto Ax = q = constant \qquad \beta = \frac{K}{Y} = \frac{1}{Ax} = constant \tag{32}$$

Ultimately, output and employment growth rates, as well as the capital/income ratio and return on capital over the long term, depend on one variable, the RCI, taking values between 0 and 1. Hence, the growth rates in output, employment and investment will be symbolized by g_Y, g_L, g_I. The stationary states are characterized by the following equations:

$$g_Y = g_I = Axi_n \qquad g_L = \frac{\propto A}{1-\propto}(2x-1)i_n \qquad \beta = \frac{K}{Y} = \frac{1}{Ax} \qquad r = q = \propto Ax \tag{33}$$
$$0 < x \leq 1 \qquad 0 < \propto < 1/2$$

Thus, for these stationary states, the growth rates of output and employment are constant over time; these are therefore steady states. The main fundamentals in the steady states are expressed simply by the PCI, the net investment rate, the RCI, and the profit share in income.

In general, the greater the share of investment engaged in additional output capacities, the stronger the growth and return on capital. In other words, the more successful entrepreneurs become in increasing returns, the higher the growth rate and the return on capital. The search for maximum return by entrepreneurs encourages them to increase the RCI.

The PCI and the net investment rate are exogenous data. The first reflects the speed of technical progress allowed by the techniques used and the institutions that accompany them. It does not therefore reflect the level of technical progress; a lagging economy could be characterized by a higher PCI than an advanced one. Net investment rate depends in particular on monetary conditions, which are not discussed here.

2.2 A LINEAR RELATIONSHIP BETWEEN OUTPUT, EMPLOYMENT AND INVESTMENT

The first salient insight of this new theory lies in the long-run linear output-employment-investment relationship that the steady states verify:

$$g_Y = \frac{1-\propto}{2 \propto} g_L + \frac{A}{2} i_n \qquad 0 < \propto < 1/2 \qquad g_Y > 0 \qquad -\frac{\propto}{1-\propto} Ai_n < g_L \leq \frac{\propto}{1-\propto} Ai_n \tag{34}$$

We have seen that this equation is valid when the rate of investment varies (see Equation 25), which makes it possible to talk more generally about a relationship between output, employment and investment. However, here we will consider an output-employment relationship when the net investment rate is considered constant.

For a given profit share in income $\propto$, the set of steady states is represented by the segment $G_0 G_{mx}$ of Figure 2. G_{mx} represents the maximum growth path (over the long term): the growth rates of output and employment are then maximal, with all new productive combinations being engaged in increasing returns. G_e represents the growth path with stable employment, the RCI being equal to 1/2.

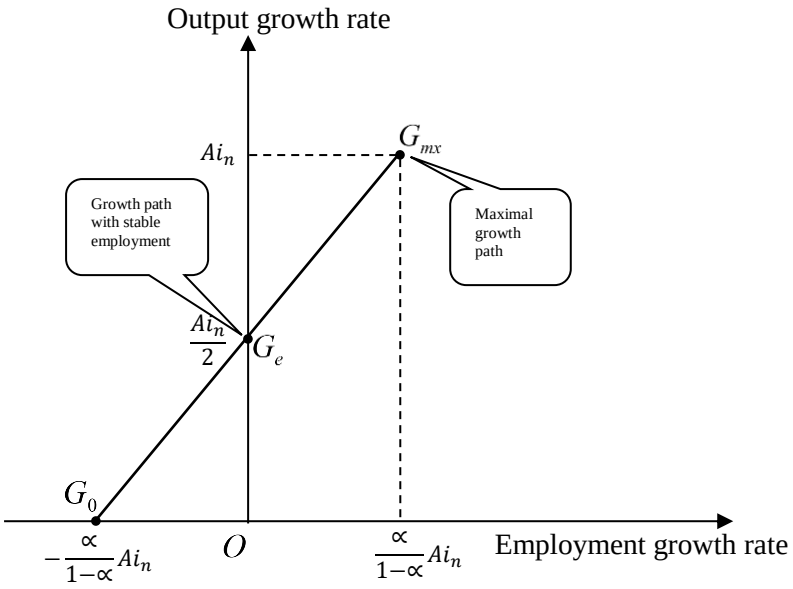

Figure 2 The relationship between output and employment

2.3 THE ECONOMIC CYCLE ATTRACTOR

In general, growth paths appear to be in imbalance, for example because competitiveness is not assured, since entrepreneurs do not have the lowest total cost per unit of output. One reason for these inadequate choices is classically derived from a dependence on the technological trajectory, as many economists have shown (e.g. David, 2000).

Given competition, these entrepreneurs are then forced to formulate adaptive expectations, in other words to adopt more competitive productive combinations or to disappear (Nelson, 2005). The return to the competitive situation represented by the steady states is necessary for these entrepreneurs; however, other entrepreneurs may be uncompetitive in the subsequent period.

The economic trajectories (annual, for example) will therefore perpetually move towards and away from the steady states, in other words from the segment G_0G_{mx} (see Figure 2). Thus, steady states have a status of "attractor" in the long term. The attractor of the steady states represents the ideal chain-reaction, the behaviors of entrepreneurs, consumers and markets being perfect. This attractor will be referred to as "the economic cycle attractor".

In the short term, economic growth above the maximum growth path can be achieved and recessions may occur; these extreme cases can be interpreted as situations where the capacity utilization rate (CUR) is temporarily increasing or decreasing, while the RCI is always between 0 and 1. Thus, the equivalent RCIs are temporarily higher than 1 or negative[11].

In addition, if the investment rate is constant, the attractor thus has a dual function over the long term: to represent the average trajectory and to attract trajectories (for example, annual trajectories). In other words, the mean values of the economic fundamentals should be those of the attractor of the steady states.

[11] In the case of a boom, job creation is very strong, while job destruction still exists, obsolete jobs being eliminated: the modeling continues to be relevant. In the case of recession, the decrease in the CUR is equivalent to investing more in rationalization; the modeling still provides for significant net job destruction.

2.4 THE PROFIT SHARE IN INCOME ON THE LONG TERM

Let the wage growth rate in relation to the employment growth rate be written:

$$g_w = g_Y - g_L = \frac{1-3\propto}{2\propto} g_L + \frac{A}{2} i_n = \frac{1-3\propto}{1-\propto} A x i_n + \frac{\propto}{1-\propto} A i_n \quad (35)$$

For a profit share in income of 1/3, wage growth is independent of both employment growth and the RCI. In other words, the wage gains in firms that grow strongly will be equal to those found in companies that grow weakly.

Thus, if the labor market operates in a perfectly homogeneous way, a wage standard is imposed on all firms, with wage gains being independent of employment growth. In this case, the profit share in income must be exactly 1/3. The profit share in income of 1/3 characterizes a distribution that will be described as "neutral", that is, a distribution that does not distort wage gains according to employment growth.

So far, economic theory has proved unable to justify a value of 1/3 (Piketty, 2014), often taken into account from the earliest modeling (Cobb-Douglas, 1928) and confirmed on average (34 %) for one set of economies at different stages of development around 1990 (Gollin, 2002)[12].

For a profit share in income other than 1/3, there is therefore a distortion of wage gains in favor or against companies which grow strongly. When the profit share in income is less than 1/3, wage gains grow at the same time as employment, making companies that create jobs very attractive. We can then assume that, in the long term, this property induces economic dynamism (an increase of the RCI) and finally a decrease in the capital/income ratio.

On the contrary, when the profit share in income is greater than 1/3, wage gains decline when employment increases, which does not make companies that create jobs attractive. Entrepreneurs' expectations in terms of job creation may then not be met. This property could lead, in the long term, to an economic slowdown (a decrease in the RCI) and finally an increase in the capital/income ratio.

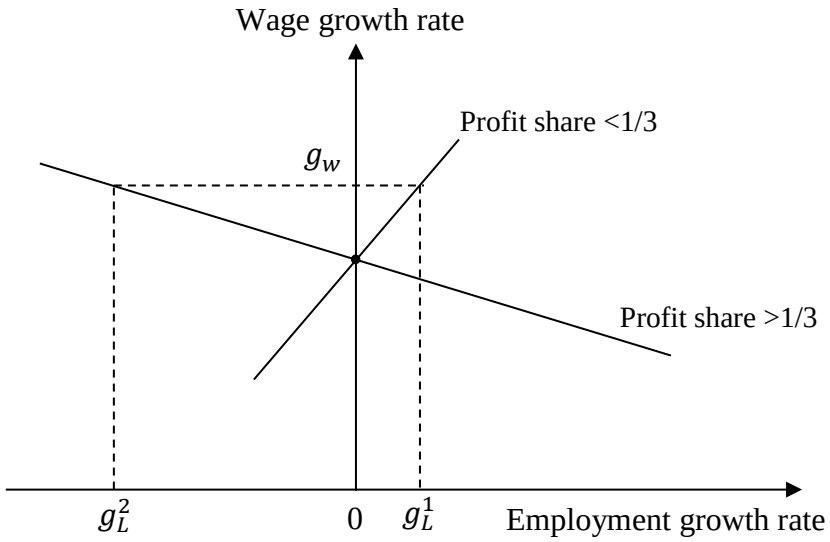

Figure 3 The economic slowdown when the profit share is above 1/3

This property is illustrated by Figure 3 which represents the evolution of the wage growth rate as a function of employment growth rate when the profit share is less than 1/3 or more than 1/3.

[12] This 34% average concerns a set of 41 countries, the profit share in income varying from 20% to 35%.

The wage rigidity, i.e. the wage growh rate is constant, leads to a negative employment growth rate when the profit share becomes higher than 1/3; thus, the output growth rate decreases.

Ultimately, the possible variations of profit share can be identified, when one takes into account the fundamental law of capitalism stated by Piketty ($r > g_Y$):

$$i_n < \propto < \frac{1}{2} \qquad in\ the\ long\ term, for\ durable\ growth:\ i_n < \propto \leq 1/3 \tag{36}$$

3. THE U.S. ECONOMY AND THE ATTRACTOR

Could this theory explain the trajectories of the U.S. economy, the largest advanced economy? To answer this question, the U.S. economy has been considered since the 1960s because precise data are available on the growth rates of GDP and employment, on rate of investment and on profit share in income.

The period 1961-2000 appears very favorable for a detailed comparison between the theoretical model and reality, given the regular and high growth observed as a trend, while rate of investment was relatively constant. Nearly full employment is encountered at the beginning and at the end of the period, which makes it possible to consider that the functioning of the labor market has been satisfactory, thus allowing the employment growth rate to adjust to the population growth rate. The profit share varied moderately, while being very close to the value of the theory of 1/3 characterizing a neutral distribution; moreover, the capital/income ratio varies very little over this period. The decade 2000 is excluded, given the occurrence of two crises, that of 2001 and that of 2008, called another Great Recession; over the decade, economic growth has declined markedly and full employment is no longer assured. The period 2001-2015 will be examined later.

3.1 AN ECONOMY CLOSE TO THE MAXIMUM GROWTH PATH

For the period 1961-2000, precise annual data (see Appendix 1) on GDP and employment growth (in hours worked) and the gross investment rate, as well as the profit share in income[13] are available. Table 1 presents the mean annual values.

Empirical fundamentals for the United States economy	1961-2000
• GDP growth rate (g_Y)	3.61 %
• Employment growth rate (g_L)	1.64 %
• Net investment rate (i_n)	15.5%
• Profit share in income $(\propto)$	34.0%

Table 1 - Fundamentals of the United States economy (1961-2000)

Based on these data, the theory presented makes it possible to calculate the mean values of the PCI and the RCI, whose equations are recalled below:

$$A = \frac{2 \propto g_Y - (1-\propto) g_L}{\propto i_n} \qquad x = \frac{\propto g_Y}{2 \propto g_Y - (1-\propto) g_L} \tag{37}$$

Table 2 presents the PCI and RCI characterizing this economy over the period 1961-2000.

[13] The data are from the World Bank (World Development Indicators-WDI-August 2016) for the GDP growth rate and the gross investment rate, from the Groningen Center for the growth rate of total hours worked (The conference Board and Groningen Growth and Development Center, Total Economy Database, August 2016, http://www.ggdc.net). Data on the profit share in income from 1961 to 2000 is taken from the European Commission (Annual macro-economic database – AMECO – May 2017). In the absence of net investment in databases, it is assumed that the proportion of replacement investment is typically 30%.

United States growth model: parameters	1961-2000
• Productivity of capacity investment (PCI)	0.260
• Ratio of capacity investment (RCI)	89.7%

Table 2 - Characteristics of the growth model of the United States economy (1961-2000)

A first striking result is the following: with an RCI of 89.7%, the average fundamentals of the economy are characteristic of the maximum growth path. In other words, the economy is, on average over the broad period 1961-2000, positioned on the path of maximum growth of output and employment, with a distribution very close to the neutral distribution of 1/3.

The second significant result comes from the fact that the United States has a profit share in income close to 1/3. Worthy of mention is the evaluation for the U.S. economy in the years 1909-1949, with an average of 34%[14] (Solow, 1957).

Are wage gains independent of employment growth, as the theory shows? For the period 1961-2000, we find no correlation between labor productivity growth and employment growth. Several economists have also made this observation over a long period[15]. For example, Salter (1960, 1966) found that there was no correlation between labor productivity gains and employment growth[16] in his survey of 27 industrial sectors of the U.S. economy from 1923 to 1950. Hansen-Wright (1992) also find that there is no correlation between labor productivity and employment. Thus, as predicted by the theory, this profit share in income close to 1/3 is clearly associated with an independence of wage gains in relation to the growth of employment.

3.2 THE ATTRACTOR

What are the output-employment relationships when assuming that the net investment rate is constant?

Linear relationship (1961-2000)		Theoretical attractor (1961-2000)	(38)
$g_Y = 0.90 g_L + 0.0214$	$R2 = 0.62$	$g_Y = 0.97 g_L + 0.0201$	

The theoretical linear relation output-employment-investment was determined from the known values of profit share in income, PCI and net investment rate. The empirical relationship is significant and the differences between the theoretical and empirical coefficients are of the order of 8%.

Figure 4 shows annual growth paths along with the theoretical attractor and the empirical linear output-employment relationship. It illustrates the unbalanced nature of annual economic growth and the role played by the attractor of the economic cycle. The trajectory of the fundamentals winds around the steady states, which then appear to play the role of attractor. It is clear that the maximum growth path (1.64%, 3.61%) is located at the top and is surrounded by a cloud of growth paths.

This figure reflects that the instabilities are, in a way, channeled around the long-term relationship characterizing the steady states. This should be seen as the impact of adaptive strategies of entrepreneurs and the result of the competitive functioning of the different markets.

The existence of a significant correlation between the annual rates of GDP growth and the annual rates of employment growth and of net investment is now checked:

[14] Annually the share varies between 31% and 40%.

[15] It is also a conclusion of Keynes: 'Workers will not seek a much greater money-wage when employment improves.' (1936, p. 253).

[16] Given the long-term consistency of the profit share in income, the wage growth rate is equal to the growth rate of labor productivity.

Linear relationship (1961-2000)	Theoretical attractor (1961-2000)	
$g_Y = 0.88g_L + 0.140i_n \quad R2 = 0.91$	$g_Y = 0.97g_L + 0.130i_n$	(39)

The annual rate of GDP growth correlates well with the annual rate of employment growth and the net investment rate. The differences between theoretical and empirical coefficients are about 10%. The output-employment-investment correlation is much more significant, although the differences are similar.

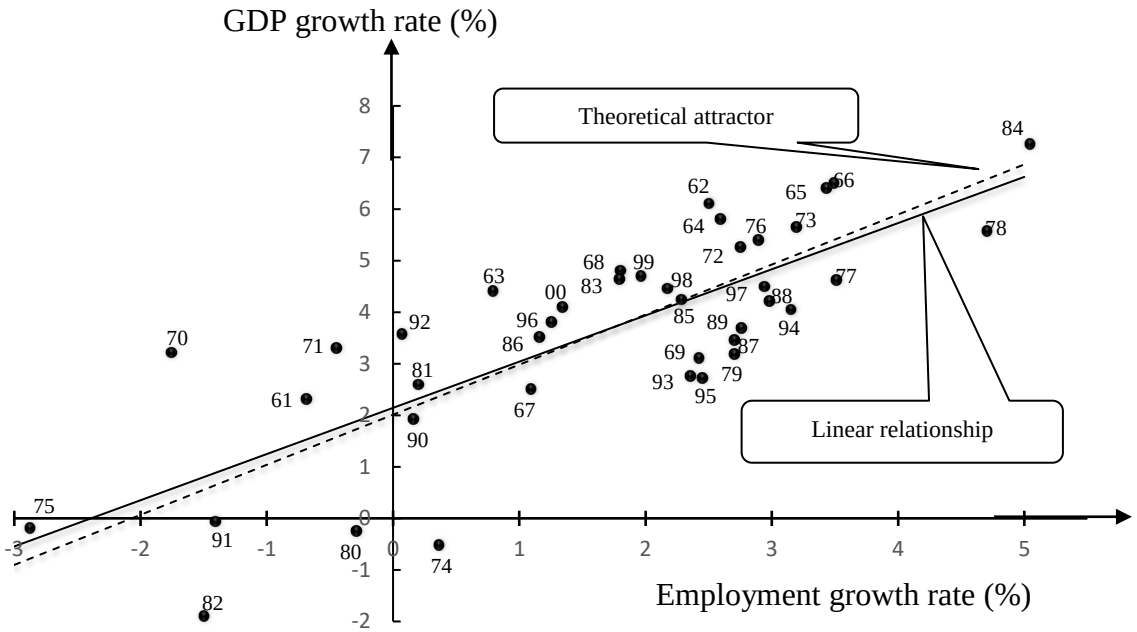

Figure 4 United Sates (1961-2000): the annual growth paths and the attractor

This finding of an output-employment coefficient of about one over a long period (equations 38 and 39) is consistent with the measurement made for different sectors and different historical periods. Indeed, Bernanke and Parkinson (1991), in the study of the evolution of output and employment in ten industries for the two periods 1924-1939 and 1955-1988, showed in the linear regressions a coefficient of employment of 1.07 and 0.96 respectively[17].

From the equation (39), we can also deduce the productivity of capacity investment of around 0.280 on average; the theory then makes it possible to predict a capital/income ratio:

$$\beta = \frac{1}{Ax} = 398\ \% \tag{40}$$

This theoretical prediction is satisfactory, the real observed capital/income ratio being around 420% over this period (Piketty, 2014)[18].

3.3 2001-2015: THE INSIGHTS

We have shown the good fit between the new growth theory and the data for the 1961-2000 period of the U.S. economy. Is it still verified for the years 2001-2015? The increase of the profit share in income since 2001 and the Great Recession of 2008-2009 introduce a rupture in economic fundamentals, which leads us to consider two periods, 2001-2008 and 2009-2015.

[17] In 72% of the cases, the coefficients for the different sectors are between 0.8 and 1.3 (based on quarterly observations).

[18] The capital / income ratio ranges from about 390% to 440% from 1961 to 2000, an average of 420% (Piketty, 2014, p. 151).

Table 3 summarizes, for the three successive periods since 1961, the macroeconomic fundamentals as well as the RIC, the PIC, the empirical linear relationship and the theoretical attractor[19]. The bursting of the stock market bubble in 2001-2002 resulted in lower GDP growth and a drop in employment growth; the GDP growth rate drops further after the Great Recession.

United-States	g_Y	g_L	i_n	$\propto$	RIC	PIC	Linear relationship and theoretical attractor
1961-2000	3.61 %	1.64 %	15.5 %	34.0 %	89.7 %	0.260	$g_Y = 0.88g_L + 0.140i_n \quad R2 = 0.91$ $g_Y = 0.97g_L + 0.130i_n$
2001-2008	2.10 %	0.14 %	15.4 %	36.3 %	53.0 %	0.257	$g_Y = 0.70g_L + 0.130i_n \quad R2 = 0.87$ $g_Y = 0.88g_L + 0.129i_n$
2009-2015	1.40 %	0.39 %	13.2 %	38.7 %	64.3 %	0.165	$g_Y = 0.66g_L + 0.087i_n \quad R2 = 0.91$ $g_Y = 0.79g_L + 0.083i_n$

Table 3 - United States from 1961 to 2015: comparison of the three periods

The first lesson is the empirical linear relationships that are still significant for the periods 2001-2008 (Figure 5) and 2009-2015 (Figure 6); however, the theoretical production-employment elasticities differ by 26% and 20%, respectively, from the empirical values.

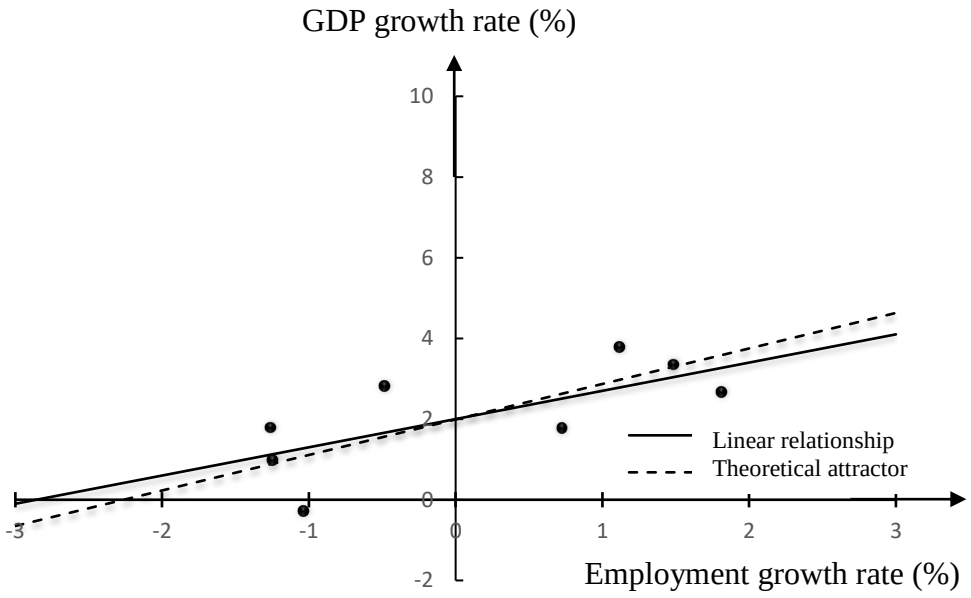

Figure 5 United States (2001-2008): the annual growth paths and the attractor

Over these two periods, the profit share in income increases sharply and rapidly, by about 2.3 points for 2001-2008 and 4.7 points for 2009-2015 compared to the period of prosperity. Most likely, this large variation in the profit share explains the larger differences between the linear relationships and the theoretical attractors for the last two periods.

[19] The same data bases are used (see 3.1).

The second lesson highlights, for the PCI, the continuity for 2001-2008 then its significant decline of the order of 36% after the Great Recession. The rupture introduced by the Great Recession is reflected in a brutal collapse of effective demand and, ultimately, that of the PCI, despite the massive stimulus policy implemented in 2009-2010.

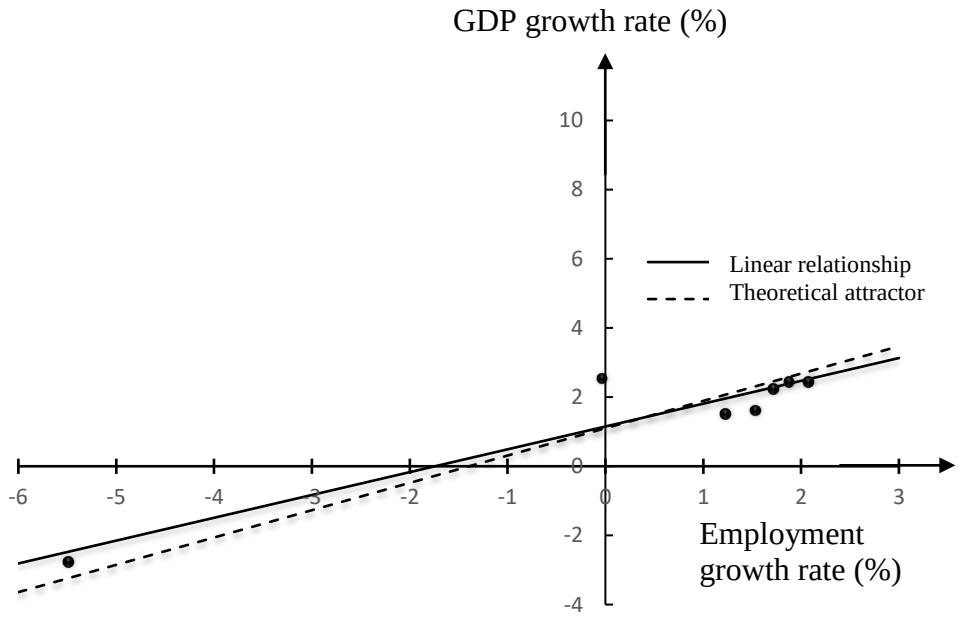

Figure 6 United States (2009-2015): the annual growth paths and the attractor

The third lesson highlights the correlation between the trend decline in the GDP growth rate and the very rapid rise in the profit share in income. Recall that the theory emphasizes that a profit share greater than 1/3 may induce long-term slowing growth. The impact of such an increase should therefore be questioned, since the long period of prosperity between 1961 and 2000 had been characterized by a profit share that was almost constant and very close to the value of 1/3 of a neutral distribution. Effective demand is probably not strong enough, especially given the rises in profits and in household savings.

4. DISCUSSION AROUND THIS NEW THEORY

4.1 A KEYNESIAN AND ENDOGENOUS MODEL OF GROWTH

This new theory rehabilitates rational expectations formulated by entrepreneurs, be it on output, employment, investment, wages, profits and, of course, on future return on investment. It thus highlights the chain-reaction constituted by successive increases in supply and demand, the chain-reaction being regulated by the marginal efficiency of capital.

Rationality is bounded, taking into account the complexity of the decisions to be made. Maximization of profit is obviously sought, but minimization of unit output cost, with a concern for long-term competitiveness, is an indispensable step, as Schumpeter had theorized.

The growth process is based on an *AK*-type endogenous growth model. However, capital *K* does not integrate "human capital" as do many models of endogenous growth. As Piketty (2014) notes, after long-term analysis of changes in the capital/income ratio and capital/labor sharing, there is no evidence of a "human capital" that should have altered these developments.

The *AK* model is combined with a process of creative destruction, as Aghion-Howitt (1998) have studied: however, it is assumed that creative destruction manifests itself through two types of investment (capacity and rationalization) and not through types of innovation.

In view of the imbalances engendered by the chain-reaction, the relative regularity of growth or the relative convergence around the attractor of the economic cycle can only be explained by the existence of very important feedback forces. The latter reside in the competitive functioning of the different markets (goods, labor, finance, etc.) which give valuable indications to entrepreneurs to formulate their adaptive strategies. These forces were not modeled in the growth model.

In general, economic development is based on the material (including software) and social technologies implemented. Investments, of a material or software nature by definition, are accompanied by intangible investments (training, organization, etc.) which are not modeled here. However, the PCI seems to indirectly reflect the productive efficiency of these intangible investments and also that of the functioning of the labor market. The maximum growth path reflects the excellence of a growth regime that combines new technologies, social technologies, innovation processes and the satisfactory functioning of the labor market.

In order to take into account many dimensions of anticipation, the model has been simplified on many non-fundamental aspects in the initial analysis. For example, the capital evolution equation has been simplified by modeling the volume of replacement investments. Investments have been categorized into the three usual categories (capacity, rationalization and replacement), while the reality may be more complex, with alternative investments that can incorporate technical progress and improve capacity. In the interests of simplification, we did not take into account the capacity utilization rate which is thus integrated into the RCI.

4.2 THE COHERENCE WITH THE STYLIZED FACTS

Via analysis of the fundamentals of the main economies of the 19th and 20th centuries, Kaldor (1961) has identified six stylized facts characterizing long-term economic growth. For Barro and Sala-i-Martin (1995, p.5) these facts are confirmed by the long-term data relative to today's developed countries: nevertheless, they added two facts.

- 1: per capita output grows over time, and its growth rate does not tend to diminish
- 2: physical capital per worker grows over time
- 3: the rate of return to capital is nearly constant
- 4: the ratio of physical capital to output is nearly constant
- 5: the shares of labor and physical capital in national income are nearly constant
- 6: the growth rate of output per worker differs substantially across countries.
- 7: a certain stability of investment and saving rates
- 8: a positive correlation between the output growth rate and the investment rate.[20]

It is easy to verify that the theoretical lessons of the growth model are potentially consistent with the stylized facts. Fact 6 is compatible with the growth model if we assume that the productivity of the capacity investment and investment rate differ from one country to another.

Is the linear theoretical relationship between output-employment-investment compatible with the empirical laws set forth by Verdoorn? Verdoorn's law (Verdoorn, 1949, 1993) estimates productivity elasticity relative to output to be close to 0.5; it is 0.484 according to Kaldor (1956).

[20] Much research supports this point of view, including De Long-Summers (1991), Levine-Renelt (1992), Bernanke-Gürkaynak (2002), Acemoglu (2009).

This has been the subject of numerous subsequent evaluations showing a wider spectrum of values; this elasticity "appears to be significantly different from both 0 and 1" (Boyer and Petit 1981: p.1117)[21].

Our theory provides an elasticity of 0.5 for an economy characterized by a maximum growth path and a neutral distribution, such as the U.S. economy (1961-2000), exactly the value found by Verdoorn and Kaldor. However, for economies characterized by other values of profit share in income or RCI, the elasticity could theoretically vary in practice from 0.5 to 1, which seems to be in line with the empirical reality.[22]

4.3 A DISTRIBUTION CONSISTENT WITH EMPIRICAL PIKETTY DATA

The profit share in income could result from the confrontation between the power of shareholders seeking the best profit and the bargaining power of unions struggling to obtain the best wage increase. Is this new theory consistent with Piketty (2014) describing the major changes in the profit share in income?

This Keynesian and endogenous growth theory gives a framework for the profit share: $i_n < \propto < 1/2$. The profit share in income for countries such as the United Kingdom and France has been between 20% and 45% since 1770 for the United Kingdom and 1820 for France. Thus, it has never been greater than 50%, which is also an upper limit for our theory.

Profit share in income is considerably greater than 1/3 between 1810 and 1870 for the United Kingdom, and between 1840 and 1870 for France, which corresponds essentially to the time of the first industrial revolution. It is also the period of Marx's analysis of industrial capitalism during which wages stagnate or even regress and profits increase. This seems to confirm the theoretical idea that a profit share in income above 1/3 may be detrimental to long-term economic growth.

Since the 1880s, the profit share in income has almost never been significantly higher than 1/3. It is well below 30% from 1920 for the United Kingdom and 1940 for France. This also seems to confirm the theoretical idea that such a profit share is generally beneficial for long-term economic growth. The 20% lower limit for the United Kingdom or France (in the 1970s and 1980s), is also consistent with the theoretical limit, with a net investment rate of around 16%.

CONCLUSION

The process of economic growth must be seen as a chain-reaction between the supply and demand increases anticipated by entrepreneurs. This process leads entrepreneurs to take decisions about production, employment and investment, in accordance with the principles of the effective demand and of the competitive supply. This chain-reaction is regulated by the marginal efficiency of capital assessed by entrepreneurs. Growth and employment trajectories, by their nature unbalanced, nevertheless admit patterns around attractor.

There are three major and unexpected insights. Firstly, output and employment growth rates and the net investment rate verify the following linear relationship:

$$g_Y = \frac{1-\propto}{2 \propto} g_L + \frac{A}{2} i_n$$

where $\propto$ is the profit share in income, i_n the net investment rate (or the net saving rate) and A the productivity of the capacity investment.

[21] Values of 0.38 to 0.93 are found, for example, for the 1961-1973 period.

[22] It is easy to demonstrate that the elasticity could increase from 0.5 to 1 (α of 0.25, x of 0.5).

Secondly, this linear relation plays the role of an attractor during the long term, with annual trajectories wrapping around this relationship, as shown by the behavior of the U.S. economy, from 1961 to 2000, a long period of prosperity. This is also the case for two differents periods, 2001-2008 and 2009-2015, with two specific linear relationships.

Thirdly, a profit share in income higher than 1/3 leads to an economic slowdown; this property is checked for the last period 2001-2015 of the U.S. economy.

These results show the great interest of a Keynesian and endogenous model combining the effective demand principle and the competitive supply principle. These results are also consistent with stylized facts highlighted by economists. In view of these results, obtained using simplified modeling, this research path appears promising.

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APPENDIX 1

	1961	1962	1963	1964	1965	1966	1967	1968	1969	1970	1971	1972	1973	1974	1975	1976	1977	1978	1979	1980
g_Y	2.3	6.1	4.4	5.8	6.4	6.5	2.5	4.8	3.1	3.21	3.3	5.26	5.64	-0.52	-0.2	5.39	4.61	5.56	3.18	-0.25
g_L	-0.68	2.51	0.8	2.6	3.44	3.5	1.1	1.81	2.43	-1.75	-0.44	2.76	3.2	0.37	-2.87	2.9	3.52	4.71	2.71	-0.28
i	21.9	22.1	22.3	22.4	22.6	22.6	22.1	22	22.9	21.2	21.2	21.8	22.2	21.7	20.6	21.1	22.4	23.7	24.4	23.5
$\propto$	31.9	32.5	32.8	33.2	34.1	34 .2	33.4	32.9	31.8	30.9	31.9	32.2	32.5	31.9	33.4	34.1	34.2	34.5	34.4	33.5

	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000
g_Y	2.59	-1.91	4.63	7.26	4.24	3.51	3.46	4.2	3.68	1.92	-0.07	3.56	2.75	4.04	2.72	3.8	4.49	4.45	4.69	4.09
g_L	0.21	-1.49	1.8	5.05	2.29	1.17	2.71	2.99	2.77	0.17	-1.4	0.08	2.36	3.16	2.46	1.26	2.95	2.18	1.97	1.35
i	23.3	22.5	22.4	23.5	23.6	23.5	23	22.4	22	21.2	20.1	19.8	20	20.3	20.8	21.3	21.5	22.2	22.7	23
$\propto$	34.3	33.5	34.9	35.6	35.7	35.3	34.7	34.3	35.1	34.6	34.2	34.2	34.8	35.5	35.8	36.2	36.1	35.1	35.2	34.2

	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015
g_Y	0.98	1.79	2.81	3.79	3.35	2.67	1.78	-0.29	-2.78	2.53	1.60	2.22	1.49	2.43	2.43
g_L	-1.24	-1.26	-0.48	1.13	1.49	1.82	0.73	-1.03	-5.48	-0.02	1.55	1.73	1.24	1.89	2.09
i	22.4	21.4	21.5	22.0	22.8	22.9	22.1	21.0	18.6	18.0	18.3	19.0	19.1	19.5	19.6
$\propto$	34.4	35.4	36.0	36.4	37.4	37.3	37.0	36.8	37.7	38.7	38.8	38.8	39.3	39.1	38.6

Table - Annual GDP growth rate, annual employment growth rate, annual gross investment rate and annual profit shares in income for 1961-2015 (in %)